

Illustration of Mathematical Imagery

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ABSTRACT Mathematics that is about logical examination and quantitative calculations is a science, which is based on abstractions. While doing abstraction, one expresses those, which are intangible and invisible, via images. An image, imagination and dream are the images appearing in the mind. Also, correlation between images is called imagery. Indeed, mathematics is an image-based work. For some, mathematics is an enjoyable work, and for others it is difficult and boring. This study was done with the aim of finding out what the secondary school students' mathematical imagery is. For this reason, a total of 46 volunteer students at secondary school were asked to illustrate mathematical imageries. These illustrations were analyzed regarding successful and unsuccessful students, the most used images were determined and three students were interviewed on the illustrations, which they drew. As a result of this study, they mostly used the images about numbers, operations and geometric figures in their illustrations.

INTRODUCTION

The image is a subjective design of the objective world (L'Abbe and Domecq 1925, cited in Isildak 2008: 64). It was derived from the Latin word *imitari* and it means imagination, and dream in Turkish, and in English it means an image appearing in psyche, a picture being reflected to mirror, copy and display image (Çetin 2011).

Descartes defined image as "*a view created by external objects, leaves traces inside the brain via senses and nerves*", and Van Gogh identified it as "*some unascertained images appearing and disappearing are wandering around in my mind*" (Isildak 2008: 65).

Imagery means creating display or non-display of ideas associating the objects that are not real but similar to real one and obtained by doing various operations on the objects which are kept in the memory, with the images gained before by a stimulus from external world or with the participation of introversive senses (Çetin 2011: 163). In other words, imagery can be defined as an ability of forming new relations between images, and creating new conceptions and images (Isildak 2008: 66).

Imagery enables to make mathematical deductions on the bases of the correlations between images. Imagery creates mathematical awareness with symbolization, association and examination. A result is reached and problems are solved. For instance, when it is known that the measurement of a triangle's interior angles is 180 degree, how can a rectangle's measurement of an interior angle be found out? For this rea-

son, how can it be calculated by drawing triangles in a rectangle by using its angles? If it were a pentagon how many triangles are drawn in it to find out? By using the image of triangle at this point, it is come to an idea on rectangular or pentagon. Even inventions can be done based on these relations between images. According to Eugene Delacroix, "the source of genius is just imagery" (Isildak 2008). Many inventions are results of creative imagery. For example, Newton who is sitting under a tree made a creative imagery "as a result of an apple's falling down" by correlating the apple image with the moon's image and he was able to conclude that a satellite's turning around the earth is "a continuous falling event". A scientist tries to prove, an artist tries to visualize the different relations that they create between images. On the basis of images such as speed, momentum and pressure, Newton revealed "creative imagery" and he thought that "light moves, so do waves, and so light can be a wave" and by this way he found out the "wave theory of light". While Archimedes was thinking about how he could find out whether the crown that was done with the king's order was made up of pure gold or while making imagery, this imagery became "creative imagery" with a "rush". Correlating the water, flown over with the bathroom bowl's sinking, with its mass, comparing it to the quantity of water overflowed by the same weight gold and finding out whether the crown is made up of pure gold or not are possible with "creative imagery" occurred by combining conceptions such as volume, mass, overflow and images in mind (Isildak 2008).

It was proved that when imagery is used to solve a mathematics problem or when an object is asked to be envisioned, all parts of the visual cortex are active (Roland and Friberg 1985).

The power of imagery in childhood is mostly stifled by the family or inside the education process. This natural competence can be developed with a deliberative, disciplined and patient effort during the individual's education process. Correlating and comparing various imageries with mathematical images can be called as mathematical imagery. Especially in mathematics classes, one uses mathematical imagery very much. For instance, if there is a steady pace between the sides of two triangles and if the angle between the two sides is the same, these two triangles can be called as the same. One uses mathematical imagery to do this. While calculating the volume of a cone, occurring by carrying a right triangle to the coordinate axis and turning it around the x-axis, one uses mathematical imagery again.

The students' mathematical images inform about how much they know the subject, their misconceptions, interest, attitude and concerns. There are written and verbal ways of finding out the mathematical imageries. Illustrating mathematical imagery is one of the written ways. This study aims at being illustrated the secondary school's 5th to 8th grade students' imageries about "mathematics" concept. It is thought that revealing the students' images about mathematics and how they associate them with other images will inform about their thoughts and attitudes towards mathematics. This will help their mathematical development. This study will reveal the underlying thoughts beliefs and feelings of students, towards mathematics.

Objective

This study aims at determining the mathematical images of the secondary school students who are successful in mathematics and those who are not successful in mathematics.

METHODOLOGY

This study in Rize/Turkey was carried out by making secondary school students draw pictures. 46 volunteer students from secondary school from 5th, 6th, 7th and 8th grades participated in this study that was done in order to find out

what the students' imageries are about mathematics conception. These students were asked to illustrate the images that they visualize and associate in their minds on mathematics. The pictures that they drew in painting lesson were examined and their descriptive analysis was done by their art teacher and the researcher. The descriptive analysis approach enables the data to be organized according to the themes revealed by the research questions and to be presented considering the questions or dimensions that were used in the interview (Yildirim and Simsek 2003). The pictures that students drew were analyzed in three different dimensions as successful at mathematics, unsuccessful at mathematics and most used images, and they were interpreted. Besides, semi-structured interviews were carried out with three students who drew the pictures that are difficult to interpret. Semi-structured interview is a technique that enables to ask detailed questions about a subject, on the condition that the answers are missing or unclear, to repeat the question, make this situation more interpretational and complete the answers (Çepni 2007).

For this study, students were given drawing paper and colored pencils. They were asked to make pictures using these papers and paintings. The pictures that they drew have been collected in mathematics lesson. After one week, video recordings and the interviews were done with some students.

First of all, the images used in the paper of students were determined. Then the use of these images and the number of frequencies was determined. Pictures of students who are successful and not successful in mathematics were divided according to their lecture grades in math. Images of students identified from the pictures divided again according to their frequency. The pictures of students were shown to expert art teachers. For unintelligible pictures, students were asked to explain.

FINDINGS

Some samples from the pictures made by the students who succeed in mathematics are presented in the figures.

When the successful students' figures are analyzed, it is seen that they drew a tree in Figure 1, hearts in Figure 3, happy people figures in Figure 5, smiley face in Figure 6 and colorful

pictures (red, orange and yellow). The golden ratio in Figure 2, radical and exponential expressions encountered in the dream world, and the Cartesian coordinate system were rarely encountered in Figures 1, 4 and 7.



Fig. 1. The picture of student. 1



Fig. 2. The picture of student. 2



Fig. 3. The picture of student. 3

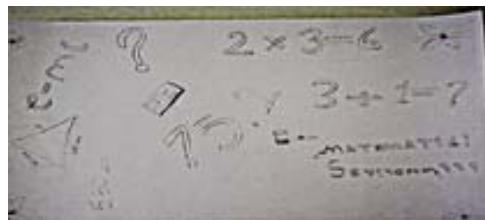


Fig. 4. The picture of student. 4

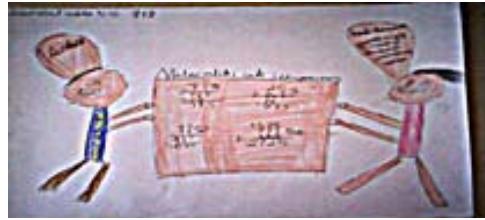


Fig. 5. The picture of student. 5

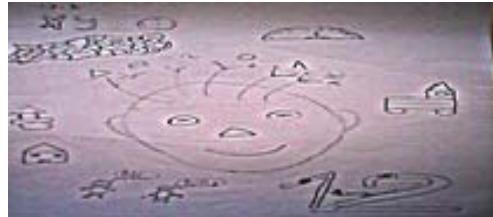


Fig. 6. The picture of student. 6



Fig. 7. The picture of student. 7

Some samples from the figures of students who were unsuccessful in mathematics are presented in the figures.

When the figures of students who are unsuccessful in mathematics were analyzed, it was seen that students drew dark, black, unhappy and confused people or figures in Figures 9, 11 and 14, and minds full of question marks in Figures 8, 12 and 13 brain and Figures 11, 13 and 15, and Azrael who claims lives in Figure 10. In some Figures, some incorrect operations (such as $8 > 9$) also emerge as in Figure 15.



Fig. 8. The picture of student. 8

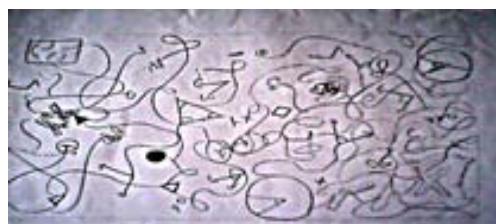


Fig. 9. The picture of student. 9

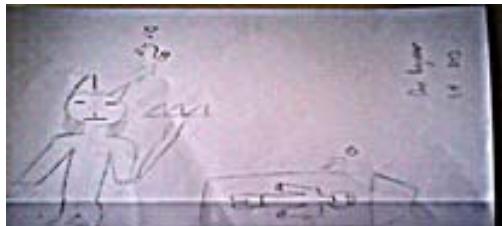


Fig. 10. The picture of student. 10



Fig. 15. The picture of student. 15



Fig. 11. The picture of student. 11



Fig. 12. The picture of student. 12



Fig. 13. The picture of student. 13

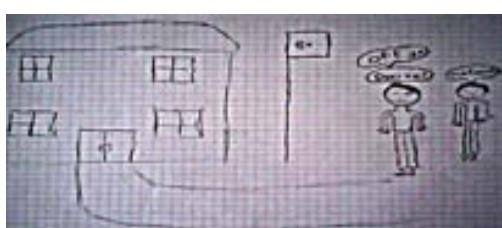


Fig. 14. The picture of student. 14

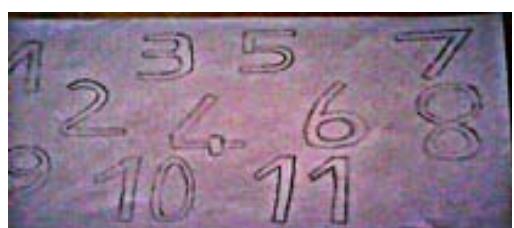


Fig. 16. Numbers

Operation: 30 (65.22%) students used operation (addition, subtraction, multiplication, division) images in their own pictures (Fig. 17).

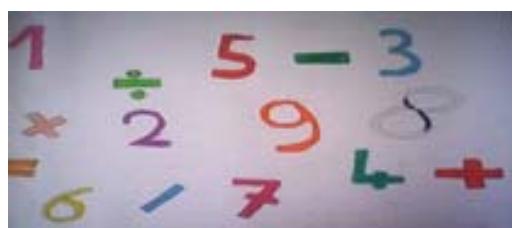


Fig. 17. Operation

Geometric figures: 17 (36.96%) students used geometric figures (square, rectangle, triangle, etc.) images in their own pictures (Fig. 18).

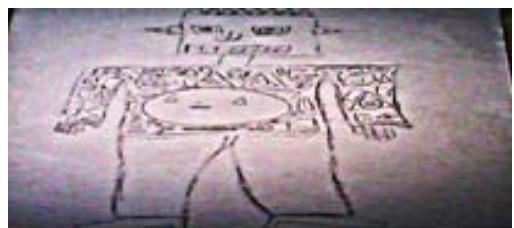


Fig. 18. Geometric figures

Question mark: 15 (32.61%) students used question mark image in their own pictures (Fig. 19).



Fig. 19. Question mark

Mathematics class: 11 (23.91%) students used mathematics class images in their own pictures (Fig. 20).

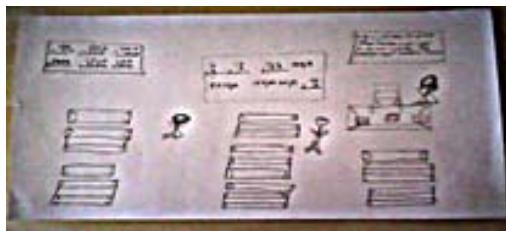


Fig. 20. Mathematics class

Pi number: 3 (6.52%) students used pi number image in their own pictures (Fig. 21).



Fig. 21. Pi number

Other mathematical images: 1 (2.17%) student used coordinate system, exponential and radical expression images in her own picture (Fig. 7).

As it is presented in pictures, students mostly used number images, then operation (addition, subtraction, multiplication, division), and later on geometric figures in their pictures. One of the rarely seen pictures was a human figure struggling with the coordinate system, exponential and radical expressions in the world of dream.

Three students whose pictures were difficult to be interpreted were interviewed and they were asked to interpret them (Fig. 22).



Fig. 22. The picture expressing mathematics anxiety

Researcher: *What does this picture mean?*

Student: *The teacher is writing a question on the board in mathematics class. The student who took turn is trying to solve it.*

Researcher: *Well, Why did the chair fall down?*

Student: *Because the question is difficult, the student is excited and dropped it.* Researcher: *Do you like mathematics?*

Student: *No.*

Researcher: *Why not?*

Student: *Because it is very difficult.*

This student, whose success level is low in mathematics, describes his anxiety with this figure he drew. In his figure, the student figure that is excited and drops the chair clearly expresses his mathematics anxiety in his mind (Fig. 21)

Researcher: *What does this picture describe?*

Student: *Mathematics is inside our lives. Here is a girl who is doing mathematics by thinking.*

Researcher: *Why is her thought shaped like heart?*

Student: *Because she likes mathematics.*

Researcher: *Do you like mathematics too?*

Student: *Yes, I do.*

Researcher: *Why?*

Student: *Because I enjoy thinking, imagining and researching.*

With her figure a student who is good at mathematics describes that she likes mathematics. She drew a student who has a heart shaped speech balloon in which there are four operations and she is solving them, and it shows love towards mathematics in her mind (Fig. 23)



Fig. 23. The picture describing mathematics love

Researcher: *What does this picture describe?*

Student: *Here is a child who likes mathematics and he is the winner, and on the other side, there is a child who doesn't like mathematics and he is the loser.*

Researcher: *Why is the one who likes a winner and the other who doesn't like a loser?*
 Student: *Because he likes mathematics he can do by heart. The one who doesn't like uses calculator. Mathematics vitalizes our lives.*

He expresses with the figure he drew that a student who is successful at mathematics considers its importance. According to him, the one who likes mathematics wins, that is to say, he becomes successful in life. However, the one who does not like it loses, that is, he cannot be successful. With the statement that "*mathematics vitalizes our lives*" he states the importance and necessity of mathematics in one's life (Fig. 24).



Fig. 24. The picture describing the importance of mathematics

DISCUSSION

Imagery flexes the mind through metaphor. Human beings live with metaphor and structure the world by it. By this way, one perceives the world perceptibly. Metaphors reflect lives, experiences and feelings (Ustaoglu 2015). In this study, students revealed their mathematical imaginations by drawing pictures with the help of metaphors reflecting their feelings. According to Piaget, pictures are the reflections of mental images on paper (Savas 2014). Thanks to the pictures that students were asked to draw, it will be possible to determine the students' feelings, ideas, fears towards mathematics and the reasons lying behind these fears, their mathematical imaginations that means how they regard it, and their misconceptions.

When the pictures drawn by the successful students are analyzed it is seen that students

drew smiley faces, happy people, hearts, trees and colorful pictures. Authorities state that the children who choose to draw with bright colors (yellow, red, orange) display affectionate, easy going and collaborative behaviors, and those who choose to draw with dull colors (purple, blue, green) display stubborn, bad-tempered and incompatible behaviors (Savas 2014). These pictures suggest how students visualize mathematics in their minds. Based on these pictures, it can be stated that students like mathematics and they have a positive attitude towards mathematics. It is pointed out in many studies that students who have a positive attitude towards mathematics become successful in mathematics (Askar and Erden 1986; Bloom 1979; Saracaloglu et al. 2004). The golden ratio, exponential and radical expressions encountered in the dream world, coordinate system, and Cartesian coordinate system were from rarely encountered pictures. It can be inferred from this fact that successful students tend to research, develop a different point of view, or since these subjects are newly learnt subjects, they were recalled from proximate memory.

When the pictures drawn by the students who are unsuccessful at mathematics are analyzed, it is seen that students drew dark, black, brain, unhappy people figures, a brain full of question marks, and Azrael claiming lives. These pictures prove that students have negative attitude towards mathematics. It has a similar result with the study that was done on mathematics metaphor and in which students who have negative attitude towards mathematics and consider mathematics as difficult and boring compare mathematics to the black color (Güveli et al. 2011). In some drawings the operations were incorrect (such as $8>9$). The researchers can also say that by this way, students' misconceptions that they have or their deficient operational or conceptual knowledge can be determined.

CONCLUSION

With this study it is enabled to express students' attitudes towards mathematics via the pictures. In the picture a student drew, the student illustrates a student who becomes excited due to the question the teacher asks and drops the chair presents his anxiety towards mathematics and a student's drawing heart shows his

love for mathematics, and a student's drawing winner and loser students proves how much he cares about mathematics. In addition, this study reveals what the students' mathematical imageries are and what kind of images come to their minds when mathematics is stated. A great deal of students' images related to mathematics consists of numbers, operations and geometrical shapes. The reason of this fact can be students being introduced with mathematics starting with numbers and going on with operations. Also, it can be said that it stems from the fact that students' mostly faced subjects, events and objects that are related to these images.

RECOMMENDATIONS

The students' images can be determined in other classes. In painting lessons, works including mathematics images, and illustrated studies in mathematics classes should be involved. In later studies, it can be studied on what the students' images related to various mathematics conceptions are and whether it is possible to find out their misconceptions or not.

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